NOTES FOR SECOND YEAR DIFFERENTIAL EQUATION PART I: INTRODUCTION AND BACKGROUND

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1. Motivations

In this course we will learn about differential equations. These are very good tools to understand dynamical systems, *i.e.*, systems which change over time. One can use a differential equation to model a system that changes continuously with time, such as the amount owned on a loan with interest compounded continuously. One can also use differential equations to model other phenomena. For instance, suppose your company produces an electronic gadget. How does the number of gadgets you sell depend on the price and the demand for your product? Can we model your sales using a difference or differential equation? We will see examples later on in the notes.

These is no general rule to find the solution of a general differential equation. Indeed, it can sometimes be a little difficult to determine **if a solution actually does exist!** Thus, we will learn a variety a methods that will work in different cases. One of the main difficulties in this course will be determining which method to use to solve a given problem. For this reason, you will need to practice using the various techniques we learn.

2. Background

We will need to recall some basic things you learned in previous courses.

Our main topic will be differential equations. For this topic we will need to recall the derivative of a function, and also partial derivative of functions of several variables. If f is a function of one variable, which we call t, then the derivative measures the rate of change of f with respect to t:

$$f'(t_0) = \left.\frac{df}{dt}\right|_{t=0} = \lim_{t\to t_0} \frac{f(t) - f(t_0)}{t - t_0}.$$

Similarly, the partial derivative of a function of several variables measures its rate of change with respect to one of its independent variables; for instance,

$$\left.\frac{\partial f}{\partial t}(s,t)\right|_{(s_0,t_0)} = \lim_{t\to t_0} \frac{f(s_0,t) - f(s_0,t_0)}{t - t_0}.$$

It would be a good idea to revise the rules of differentiation from MAM1000, as well as the rules of integration, particularly the Fundamental Theorem of Calculus and integration by parts.

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It would also be a good idea to revise the linear algebra course you just took. In particular, we will need to compute eigenvalues and eigenvectors in this course, and work with inner prodcuts. We will also use some basic properties of vector spaces, dimension, and bases.

3. Other sources

There are many good books on differential equations. I particularly like *Elementary Differential Equations* by Boyce and DiPrima, and *Differential Equations, Dynamical Systems, and Linear Algebra* by Hirsch and Smale. Both of these books are available for three hour loan in the main library, from the short loans desk. You can find other relevant books in the main library, under the call number 515.35; you're encouraged to page through several of them until you find one you like.