Introduction to and Notation for Matrix Algebra

In the first part of MAM1044, we'll talk about the algebra of matrices. An $m \times n$ matrix is rectangular arrangement of numbers with m horizontal rows an n vertical columns. For instance, a 2×3 matrix could look like

$$\begin{bmatrix} 3 & -2 & 0 \\ 1 & 4 & 2 \end{bmatrix}$$

and a 4×2 matrix could look like

$$\begin{bmatrix} 3 & -1 \\ 0 & 2 \\ -5 & 10 \\ -872 & 1 \end{bmatrix}.$$

If A in an $m \times n$ matrix, we write the number in the ith row, jth column as A_{ij}, and call it the ith, jth entry of A. For instance, in the last matrix we just wrote, we have $A_{21} = 0$.

We'll see later on that we can think of an $m \times n$ matrix transforming vectors in $\mathbb{R}^n$ into vectors in $\mathbb{R}^m$, so we may as well recall how to write these vectors. A vector in $\mathbb{R}^n$ is an ordered list of n numbers, which we can either write as a horizontal row or a vertical column. It's sometimes more convenient to write a vector as a row, and sometimes more convenient to write it as a column; the two ways of writing a vector are equivalent. For instance, we can write a vector in three-dimensional space $\mathbb{R}^3$ as

$$v = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix},$$

or we can write the same vector as

$$v = \begin{bmatrix} -1 & 0 & 2 \end{bmatrix}.$$

You may have also seen the same vector written as $v = (-1, 0, 2)$, which is a third, equivalent way to write vectors. Different people will have different ways of writing a vector, and also use special ways to denote that a letter stands for a vector. For instance, some people will write a little arrow over v if it's supposed to be a vector, and some will underline v if it's supposed to be a vector, and some will put v in boldface if it's supposed to be a vector. Different people will have different conventions for writing vectors, so be careful to understand what an author means when you read books. In class, we'll just use normal letters to write a vector (or a matrix), and we'll write a vector either as a row or a column, depending on what we want to do. If we write the vector v as a row, say $v = (v_1, v_2, \ldots, v_n)$, and $1 \leq j \leq n$, we call the nnumber v_j the jth component, or the jth entry, of v.

We should also disscuss what large dimensional spaces are. For instance, you can recognize $\mathbb{R}^2$ as a usual flat plane and $\mathbb{R}^3$ as the usual space you live in, but imagining $\mathbb{R}^5$ is a little more difficult. It's easier to think of the dimensions as parameters you get to choose, or degrees of freedom. For instance, the price of a liter of petrol might depend on four different things: supply, demand, taxes, and import duties. So, a plot of the price of petrol would be an arrangement of points in five-dimensional space, with the first four components being supply, demand, taxes, and import duties, and the last component being the price.

Exercise: Think of something else you know from every day life that depends on four (or more) parameters.

Later on we'll also want to write down vectors and matrices are complex numbers. In this case, we write n-dimensional vectors v with complex entries as $v \in \mathbb{C}^n$. Don't worry if you haven't seen complex numbers before, we'll review them when the time comes.